MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/23 (TERM ID 2215)

EMT1016 – ENGINEERING MATHEMATICS 1 / ERM1036 – ENGINEERING CALCULUS

13th FEBRUARY 2023

2.30 PM – 4.30 PM

(2 hours)

INSTRUCTIONS TO STUDENTS:

1. This exam paper consists of **6 pages** (including cover page) with **4** Questions only.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of marks for each question is given.
3. Please write your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.
4. Only NON-PROGRAMMABLE calculator is allowed.
5. Some useful formulae and tables are provided in the **Appendix** for your reference.

Question 1

(a) Evaluate the limits, if they exist.

(i) $\lim_{x\to 0^+} \dfrac{\sqrt{x^2+25}-5}{x}$ [4 marks]

(ii) $\lim_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right)$ [4 marks]

(b) The equation of a curve is given by $x^2+y^3-2x+12y=12$. Using implicit differentiation, find $\dfrac{dy}{dx}$ in terms of x and y. Hence, find the point on the curve such that its tangent is parallel to the x-axis. [6 marks]

(c) Using long division and partial fraction decomposition, show that the function $\dfrac{x^3+1}{x^2-x}$ can be decomposed to the following form

$$\frac{x^3+1}{x^2-x}=x+1+\frac{A}{x}+\frac{B}{x-1},$$

where A and B are constants. Hence, or otherwise, evaluate $\int \dfrac{x^3+1}{x^2-x}dx$. [6 marks]

(d) Find the area of the region bounded by the curves $y=e^x$, $y=e^{-x}$, $x=0$, and $x=1$. [5 marks]

Question 2

(a) Find the radius of convergence for $\sum_{n=1}^{\infty}\dfrac{(3x-5)^n}{5^n}$. [6 marks]

(b) Given that $\dfrac{1}{1-x}=\sum_{n=0}^{\infty}x^n$ for $|x|<1$, evaluate $\dfrac{1}{3x-5}$ as a Maclaurin series.

[4 marks]

(c) Consider the complex number $z=2-2\sqrt{3}i$.

(i) Find the modulus and principal argument of z. [4 marks]

(ii) Use De Moivre's Theorem to calculate z^4. [3 marks]

(iii) Find all the four roots of $z^{\frac{1}{4}}$. [5 marks]

(iv) Evaluate $Ln(z)$. [3 marks]

Continued...

Question 3

(a) A periodic function $g(x)$ has the following Fourier series representation:

$$\frac{40}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\frac{n\pi x}{4}.$$

(i) Determine the period of $g(x)$. [2 marks]

(ii) Is $g(x)$ an even function, odd function, or neither? Justify your answer. [2 marks]

(b) Suppose $g(x)$ in part (a) is multiplied with another periodic function $h(x)$ shown in **Figure Q3** below:

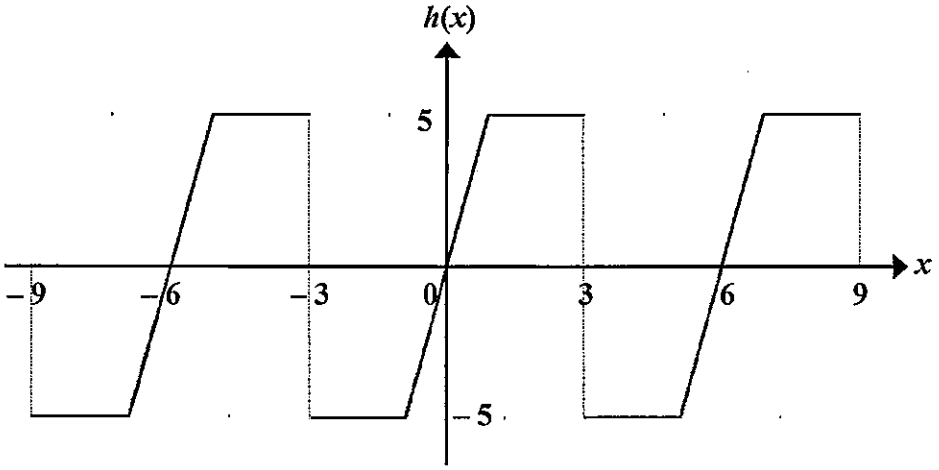


Figure Q3

(i) Would $g(x)\times h(x)$ be odd, even or neither? Justify your answer. [2 marks]

(ii) What would be the period of $g(x)\times h(x)$? [2 marks]

(c) A function $f(x)$ is defined over the interval $0 \le x \le 2$ by

$$f(x) = 2 - x.$$

Suppose $f(x)$ is an **even** periodic function with **period 4**:

(i) Sketch the graph of $f(x)$ from $x = -6$ to $x = 8$. [4 marks]

(ii) Construct a trigonometric Fourier series to represent $f(x)$. [13 marks]

Continued...

Question 4

(a) A continuous random variable, X, has probability density function

$$f(x)=\begin{cases}k(x+2), & -2\le x\le 2,\\ 0, & \text{otherwise.}\end{cases}$$

(i) Show that $k=\frac{1}{8}$. [3 marks]

(ii) Evaluate the mean of X. Hence, find $P(X>\mu)$. [5 marks]

(**Note**: For solving parts (b) and (c) below, you may use the tables in APPENDIX A and APPENDIX B.)

(b) The diameters of screws produced by a particular machine follow a normal distribution with mean 1.35 cm and standard deviation 0.04 cm. Screws with diameters less than 1.25 cm are rejected for being too small, while those with diameters more than d cm are rejected for being too large.

(i) Estimate the proportion of screws that are too small. [5 marks]

(ii) Determine the value of d if 99% of the screws have acceptable diameters. [5 marks]

(c) A customer service center receives calls at an average rate of 1 call every 10 minutes.

(i) Compute the probability that 2 or more calls are received in the next 10 minutes. [4 marks]

(ii) If the operators of the service center break for lunch from 12.30p.m. – 1.00p.m., what is the probability that they do not miss any call? [3 marks]

Continued...

APPENDIX A: Special Probability Distributions

Binomial Distribution, $X \sim b(x,n,p)$

$P(X=x) = {}^{n}C_{x}p^{x}q^{n-x}$, for $x = 0,1,...,n$ and where $q = 1-p$.

$E[X] = np$; $Var(X) = npq$

Poisson Distribution, $X \sim p(x,\lambda)$

$$P(X=x) = \frac{\lambda^{x}e^{-\lambda}}{x!}, \text{ for } x = 0,1,....$$

$E[X] = \lambda$; $Var(X) = \lambda$

Continuous Uniform Distribution, $X \sim U(a,b)$

$$f_X(x) = \begin{cases} \dfrac{1}{b-a}, & a < x < b \\ 0, & \text{elsewhere} \end{cases}$$

$$E[X] = \frac{a+b}{2}; \ Var(X) = \frac{(b-a)^2}{12}$$

Exponential Distribution, $X \sim \exp(1/\beta)$

$$f_X(x) = \begin{cases} \dfrac{1}{\beta}e^{-x/\beta}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

$E[X] = \beta$; $Var(X) = \beta^2$

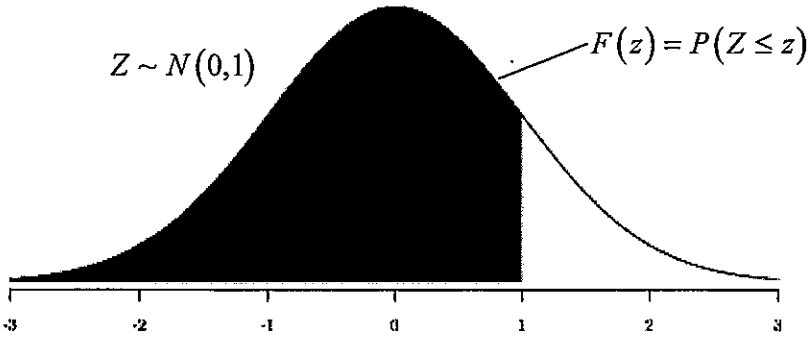
Normal Distribution, $X \sim N(\mu,\sigma)$

If X is any normal random variable where $X \sim N(\mu,\sigma)$, then the transformation

$$Z = \frac{X-\mu}{\sigma}$$

yields a *standard* normal variable where $Z \sim N(0,1)$.

APPENDIX B: Cumulative Standard Normal Distribution



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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